

A -Dependence of $\Lambda\Lambda$ bond energies in double- Λ hypernuclei

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Abstract

The A -dependence of the bond energy $\Delta B_{\Lambda\Lambda}$ of the $\Lambda\Lambda$ hypernuclear ground states is calculated in a three-body $\Lambda + \Lambda + {}^A Z$ model and in the Skyrme-Hartree-Fock approach. Various $\Lambda\Lambda$ and Λ -nucleus or ΛN potentials are used and the sensitivity of $\Delta B_{\Lambda\Lambda}$ to the interactions is discussed. It is shown that in medium and heavy $\Lambda\Lambda$ hypernuclei, $\Delta B_{\Lambda\Lambda}$ is a linear function of r_{Λ}^{-3} , where r_{Λ} is rms radius of the hyperon orbital. It looks unlikely that it will be possible to extract $\Lambda\Lambda$ interaction from the double- Λ hypernuclear energies only, the additional information about the Λ -core interaction, in particular, on r_{Λ} is needed.

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1 Introduction

Up to now, theoretical studies of double- Λ hypernuclei were mainly focused on the light systems, for only ${}_{\Lambda\Lambda}^6\text{He}$, ${}_{\Lambda\Lambda}^{10}\text{Be}$ [1], and ${}_{\Lambda\Lambda}^{13}\text{B}$ [2] have been identified experimentally. The key quantity of a $\Lambda\Lambda$ hypernucleus is the bond energy of two Λ 's, $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$, where $B_{\Lambda\Lambda}$ is the separation energy of two Λ 's from a ${}_{\Lambda\Lambda}^{A+2}Z$ hypernucleus and B_{Λ} is the Λ separation energy in the corresponding single- Λ hypernucleus ${}_{\Lambda}^{A+1}Z$. The bond energy is conventionally associated with the matrix element of $\Lambda\Lambda$ interaction in the 1S_0 state.

It is known that for light $\Lambda\Lambda$ hypernuclei, the bond energy is nearly constant, $\Delta B_{\Lambda\Lambda} \sim 4.5$ MeV. However, the respective information on $\Lambda\Lambda$ interaction derived from this value appears to be rather ambiguous due to substantial uncertainties in the shapes of both $\Lambda\Lambda$ and ΛA potentials. Using various ΛA potentials that provide the same B_{Λ} values but different Λ -orbital radii r_{Λ} , one derives very different predictions for the $\Lambda\Lambda$ interaction [3, 4, 5]. For example, the strength of $\Lambda\Lambda$ potentials of the same shape extracted from the experimental $\Delta B_{\Lambda\Lambda}$ value for ${}_{\Lambda\Lambda}^6\text{He}$ using various $\Lambda\alpha$ potentials, differ by a factor of several times [5]. If the $\Lambda\Lambda$ potential is supposed to be purely attractive, then the less is r_{Λ} the larger is $\Lambda\Lambda$ attraction. Nevertheless, one can believe, that the information about the $\Lambda\Lambda$ interaction deduced from the $\Lambda\Lambda$ hypernuclear data, will become less ambiguous if a larger number of $\Lambda\Lambda$ hypernuclei differing essentially in masses, will be involved in the analysis.

In view of evolving (K^- , K^+) experiments [6, 7], we present calculations of $\Delta B_{\Lambda\Lambda}$ for $\Lambda\Lambda$ hypernuclear ground states on a wide range of A . As far as we know, the A -dependence of $\Delta B_{\Lambda\Lambda}$ has not been studied systematically. The motivation of our study is two-fold. First, we investigate the general trend of the A -dependence of $\Delta B_{\Lambda\Lambda}$. Next, we aim to examine how much is the difference in the $\Delta B_{\Lambda\Lambda}$ predictions obtained in distinct models using various $\Lambda\Lambda$ potentials, and whether it will be possible to obtain the definite information about the $\Lambda\Lambda$ interaction from future experiments. We calculate the bond energies in two essentially different approaches. In particular, we discuss below the results obtained in the three-body model $\Lambda + \Lambda + {}^AZ$ with various $\Lambda\Lambda$ and ΛA potentials in comparison with the results of microscopic Hartree-Fock calculations with Skyrme-like $\Lambda\Lambda$, ΛN , and NN interactions.

2 Potentials and models

Within the three-body approach, we treat a $\Lambda\Lambda$ hypernucleus ${}_{\Lambda\Lambda}^{A+2}Z$ as a $\Lambda + \Lambda + {}^AZ$ system with the inert core AZ . The conventional variational oscillator basis expansion method has been used in calculations. The oscillator frequency $\hbar\omega$ has been varied independently for each hypernucleus to minimize the ground state energy. Typically the complete $22\hbar\omega$ configuration space has been used in calculations that provided the convergence of the results. The convergence has been also checked by the numerical location of the S -matrix pole corresponding to the ground state

using the technique of the harmonic oscillator representation of the true three-body scattering theory [8] (see also [9]).

The ΛA interaction has been described by the potentials proposed in ref. [10] fitted to the single- Λ hypernuclear spectra from (π^+, K^+) reaction in the range of masses $A = 8 \div 88$ [11]. The potentials are also compatible with the recent data on $A = 138$ and 207 hypernuclei [12]. The first potential (hereafter referred to as C1) is of the Woods-Saxon form:

$$\text{C1 : } \begin{cases} V_{C1}(r) = V_0 f(r), & f(r) = \left(1 + e^{(r-c)/a}\right)^{-1}; \\ V_0 = -28 \text{ MeV}, & c = (1.128 + 0.439 A^{-2/3}) A^{1/3} \text{ fm}, \\ a = 0.6 \text{ fm}. \end{cases} \quad (1)$$

The second potential (C2) is the local potential of the ρ^2 density-dependent form [10]:

$$\text{C2 : } \begin{cases} V_{C2}(r) = V_1 \rho(r) + V_2 \rho^2(r), & \rho(r) = \rho_0 f(r); \\ V_1 = -340 \text{ MeV}, & V_2 = 1087.5 \text{ MeV}, \\ c = 1.08 A^{1/3} \text{ fm}, & a = 0.54 \text{ fm}, \end{cases} \quad (2)$$

where ρ_0 is a factor normalizing the nuclear density $\rho(r)$ to A .

Single- Λ states generated by the potentials, have nearly the same B_Λ values but differ in the rms radii r_Λ .

As for the $\Lambda\Lambda$ interaction, we used, firstly, the simplest single-Gaussian potential with the range motivated by the two-pion exchange model, that is widely used [13] in double- Λ hypernuclei calculations (hereafter referred to as $\Lambda 1$):

$$\Lambda 1 : \begin{cases} V_{\Lambda 1}(r) = V_0 \exp\left(-\frac{r^2}{r_0^2}\right); \\ r_0 = 1.034 \text{ fm}. \end{cases} \quad (3)$$

The second $\Lambda\Lambda$ potential ($\Lambda 2$) used is a two-Gaussian parametrization of ref. [14] with a repulsive core:

$$\Lambda 2 : \begin{cases} V_{\Lambda 2}(r) = V_1 \exp\left(-\frac{r^2}{r_1^2}\right) + V_2 \exp\left(-\frac{r^2}{r_2^2}\right); \\ V_1 = 148 \text{ MeV}, & r_1 = 0.82 \text{ fm}, & r_2 = 1.2 \text{ fm}. \end{cases} \quad (4)$$

We have fitted the $\Lambda 1$ potential depth, V_0 , and the attractive component strength of the $\Lambda 2$ potential, V_2 , to the bond energy of ${}^{13}_{\Lambda\Lambda}\text{B}$ ($\Delta B_{\Lambda\Lambda} = 4.8 \text{ MeV}$ [2, 15, 16]) for each of the ΛA potentials independently. For boron, the ΛA potentials considered give nearly the same single- Λ orbital radii r_Λ . Therefore, we have obtained the values $V_0 = -62 \text{ MeV}$ and $V_2 = -107 \text{ MeV}$ adjusted to both C1 and C2 potentials. The differences between the C1 and C2 radii r_Λ are pronounced in medium and heavy hypernuclei.

We have performed also Hartree-Fock calculations of the bond energies with Skyrme-like NN , ΛN , and $\Lambda\Lambda$ potentials. The Skyrme-Hartree-Fock approach has been applied before to the single- Λ hypernuclei, and Skyrme ΛN potentials have been fitted to single- Λ hypernuclear spectra [10, 17, 18, 19]. This approach has been extended to multi- Λ systems in ref. [20]. Its adaptation to double- Λ hypernuclei is described elsewhere [21].

$\Lambda\Lambda$ interaction in the ground states was described by a simplified version of the Skyrme potential [21],

$$V(\vec{r}_1, \vec{r}_2) = \lambda_0 \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2} \lambda_1 \left[\vec{k}'^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2) \vec{k}'^2 \right], \quad (5)$$

with the conventional notations (see, e.g., [17]). the were used in calculations:

$$S\Lambda\Lambda 1 : \quad \lambda_0 = -312.6 \text{ MeV} \cdot \text{fm}^3, \quad \lambda_1 = 57.5 \text{ MeV} \cdot \text{fm}^5; \quad (6)$$

$$S\Lambda\Lambda 2 : \quad \lambda_0 = -437.7 \text{ MeV} \cdot \text{fm}^3, \quad \lambda_1 = 240.7 \text{ MeV} \cdot \text{fm}^5; \quad (7)$$

$$S\Lambda\Lambda 3 : \quad \lambda_0 = -831.8 \text{ MeV} \cdot \text{fm}^3, \quad \lambda_1 = 922.9 \text{ MeV} \cdot \text{fm}^5. \quad (8)$$

The sets have been fitted to the ${}^{13}_{\Lambda\Lambda}\text{B}$ bond energy in ref. [22]. The $S\Lambda\Lambda 2$ interaction is a Skyrme-like approximation of the single-Gaussian potential of the two-pion-exchange range while the $S\Lambda\Lambda 1$ interaction corresponds to a rather smaller range. The $S\Lambda\Lambda 3$ interaction possesses the largest range.

The Hartree-Fock calculations also involved SkM^* NN interaction. As for the ΛN interaction, we mainly used YBZ5 potential [18]. This potential causes only a slight polarization (distortion) of a nuclear core. The majority of the ΛN potentials fitted to single- Λ hypernuclear spectra possess this feature [10, 18, 19] though the case of the relatively strong polarization cannot be excluded now.

3 Results and discussion

The A -dependence of $\Delta B_{\Lambda\Lambda}$ is depicted on fig. 1. It is seen that in general $\Delta B_{\Lambda\Lambda}$ decreases with A increasing, although for small A it is known to be practically constant [1, 2, 16]. Attraction between hyperons become weaker just because of the increase of the mean $\Lambda\Lambda$ distance with A . In the limit $A \rightarrow \infty$, evidently, $B_{\Lambda\Lambda} \rightarrow 2D_\Lambda$ and $\Delta B_{\Lambda\Lambda} \rightarrow 0$, where D_Λ is the binding energy of the hyperon in the infinite nuclear matter.

The differences between $\Delta B_{\Lambda\Lambda}$ values obtained in the same approach with different potentials are not very large, as well as ones obtained by the two essentially different methods. We have examined $\Delta B_{\Lambda\Lambda}$ in the three-body approach using also other $\Lambda\Lambda$ potentials collected in ref. [5]; the results confirm the above conclusion. For the same $\Lambda\Lambda$ potential, $\Delta B_{\Lambda\Lambda}$ increases with the range of the $\Lambda\Lambda$ interaction. It is clearly seen that only essential variations of potential parameters (the $S\Lambda\Lambda 3$

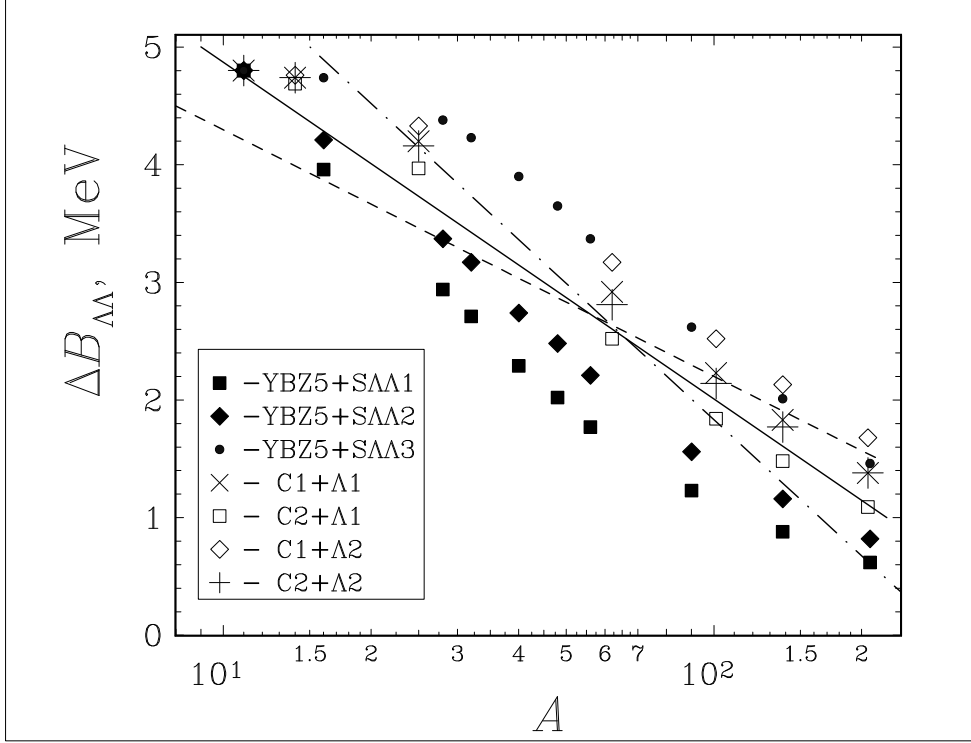


Figure 1: $\Delta B_{\Lambda\Lambda}$ as a function of A . The results of the Skyrme-Hartree-Fock calculations with ΛN potential YBZ5 and various $\Lambda\Lambda$ interactions of ${}^{13}_{\Lambda\Lambda}\text{B}$, ${}^{18}_{\Lambda\Lambda}\text{O}$, ${}^{30}_{\Lambda\Lambda}\text{Si}$, ${}^{34}_{\Lambda\Lambda}\text{S}$, ${}^{42}_{\Lambda\Lambda}\text{Ca}$, ${}^{50}_{\Lambda\Lambda}\text{Ca}$, ${}^{58}_{\Lambda\Lambda}\text{Ni}$, ${}^{92}_{\Lambda\Lambda}\text{Zr}$, ${}^{140}_{\Lambda\Lambda}\text{La}$, and ${}^{210}_{\Lambda\Lambda}\text{Pb}$ are presented by: \blacksquare — YBZ5+S $\Lambda\Lambda$ 1, \blacklozenge — YBZ5+S $\Lambda\Lambda$ 2, \bullet — YBZ5+S $\Lambda\Lambda$ 3; the results of the three-body calculations with various ΛA and $\Lambda\Lambda$ interactions of ${}^{13}_{\Lambda\Lambda}\text{B}$, ${}^{16}_{\Lambda\Lambda}\text{C}$, ${}^{27}_{\Lambda\Lambda}\text{Na}$, ${}^{64}_{\Lambda\Lambda}\text{Co}$, ${}^{103}_{\Lambda\Lambda}\text{Ru}$, ${}^{140}_{\Lambda\Lambda}\text{La}$, and ${}^{208}_{\Lambda\Lambda}\text{Hg}$ are presented by: \times — C1+ Λ 1, \square — C2+ Λ 1, \diamond — C1+ Λ 2, $+$ — C2+ Λ 2. Solid, dashed, and dot-dashed lines are the plots of the functions $\Delta B_{\Lambda\Lambda} = 15A^{-1/2}$ MeV, $\Delta B_{\Lambda\Lambda} = 9A^{-1/3}$ MeV, and $\Delta B_{\Lambda\Lambda} = 75A^{-1}$ MeV, respectively.

interaction versus the $S\Lambda\Lambda$ 1 and $S\Lambda\Lambda$ 2 ones) provides more or less considerable differences in the bond energies.

It is seen that the A -dependence of $\Delta B_{\Lambda\Lambda}$ in $\Lambda\Lambda$ hypernuclei is not very sensitive to the details of the interactions and hence cannot provide unambiguous information about the $\Lambda\Lambda$ potential strength and shape.

To get the estimate of the effects that the core polarization may have on $\Delta B_{\Lambda\Lambda}$, we have performed also the Skyrme-Hartree-Fock calculations with two other ΛN potentials: the SkSH1 [19] and the 3rd potential from [17] (hereafter R3). The potentials cause the strong polarization of the core and can be treated as two extreme examples of what can happen with a nucleus when a Λ hyperon is added to it: the SkSH1 interaction results in contraction while the R3 in dilatation of the core. Using strongly polarizing potentials one should renormalize (sometimes considerably) the $\Lambda\Lambda$ interaction to reproduce the boron bond energy [21], namely, the parameters

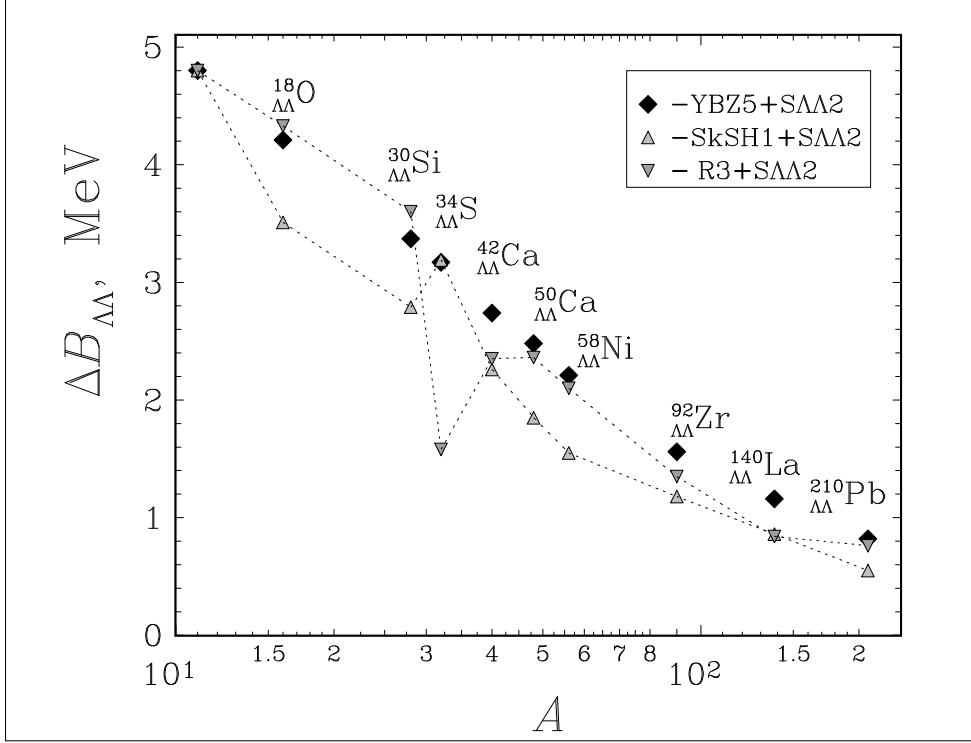


Figure 2: The A -dependence of $\Delta B_{\Lambda\Lambda}$ obtained in the Skyrme-Hartree-Fock approach with diluting R3 and contracting SkSH1 polarizing ΛN -interactions in comparison with the results obtained with slightly polarizing YBZ5 ΛN -interaction: \blacklozenge — YBZ5+S $\Lambda\Lambda$ 2, \blacktriangle — SkSH1+S $\Lambda\Lambda$ 2, \blacktriangledown — R3+S $\Lambda\Lambda$ 2. Dotted lines connecting the points are plotted to guide the eye. See fig. 1 for more details.

λ_0 and λ_1 of the S $\Lambda\Lambda$ 2 potential should be multiplied by a factor of 0.33 in the case of SkSH1 interaction and by a factor of 1.09 in the case of the R3 one.

The results of Skyrme-Hartree-Fock calculations with the strongly polarizing ΛN interactions are presented on fig. 2. It is seen that the core polarization does not change the general trend of the bound energy A -dependence. However, it results in some irregularities of the A -dependence. The irregularities are associated with some details of the ΛN interactions. For example, adding four s -nucleons to $^{30}_{\Lambda\Lambda}\text{Si}$ results in reduction of the Λ -orbital rms radius r_Λ in $^{34}_{\Lambda\Lambda}\text{S}$ for the purely two-body SkSH1 potential. Therefore, the $\Lambda\Lambda$ attraction increases. Contrary to it, in the case of the R3 interaction with strong ΛN repulsion, the hyperons in $^{34}_{\Lambda\Lambda}\text{S}$ are pushed out from the nuclear interior by the additional s nucleons, thus, r_Λ increases and $\Delta B_{\Lambda\Lambda}$ decreases. The reverse effect takes place when 8 f nucleons are added to $^{42}_{\Lambda\Lambda}\text{Ca}$ and $^{50}_{\Lambda\Lambda}\text{Ca}$ forming $^{50}_{\Lambda\Lambda}\text{Ca}$ and $^{58}_{\Lambda\Lambda}\text{Ni}$, respectively. Note, however, that ^{32}S and ^{56}Ni are not actually magic nuclei, so, the A -dependence of the occupation numbers and, hence, of $\Delta B_{\Lambda\Lambda}$ may be smoothed in more elaborate calculations.

$\Delta B_{\Lambda\Lambda}$ appears to be nearly linear function of $\log A$ for medium and heavy double-

Λ hypernuclei, i.e., $\Delta B_{\Lambda\Lambda}$ is proportional to A^α . To interpret this behavior, let us consider the following toy model. Suppose that the Λ -core potential $V_{\Lambda A}(r)$ satisfies the scaling property,

$$V_{\Lambda A}(r) = V_{\Lambda A}^0 f(r'), \quad r = r' R_A, \quad (9)$$

i.e., the strength of the potential, $V_{\Lambda A}^0$, is the same for various heavy nuclei, and the r -dependence of $V_{\Lambda A}(r)$ is described by some universal function $f(r')$ of the scaled distance $r' = r/R_A$. The scaling property (9) is exact, e.g., for the harmonic oscillator and rectangular well potentials.

The potential scaling (9) implies the scaling of the Λ hyperon radial wave function in the single- Λ hypernuclei ${}^{A+1}_{\Lambda\Lambda}Z$,

$$\phi_\Lambda^A(r) = N_A \varphi_\Lambda(r'), \quad (10)$$

where $\varphi_\Lambda(r')$ is the universal hyperon wave function and the normalization constant $N_A \sim R_A^{-3/2}$. We suppose that $\Lambda\Lambda$ interaction, $V_{\Lambda\Lambda}(r)$, is weak enough and does not perturb significantly the single- Λ orbitals (10) in the $\Lambda\Lambda$ hypernucleus ${}^{A+2}_{\Lambda\Lambda}Z$. If the range of $V_{\Lambda\Lambda}(r)$ is small compared with the rms radius r_Λ of the orbital (10), then for evaluation purposes we can treat $V_{\Lambda\Lambda}(r)$ as a zero-range potential, $V_{\Lambda\Lambda}(r) = V_{\Lambda\Lambda}^0 \delta(r)$, where $\delta(r)$ is the Dirac δ -function. In this case, we immediately obtain, that

$$\Delta B_{\Lambda\Lambda} = |\langle V_{\Lambda\Lambda}(r) \rangle| = V_{\Lambda\Lambda}^0 R_A^{-3} \langle \varphi^4(r') \rangle, \quad (11)$$

where the universal matrix element $\langle \varphi^4(r') \rangle$ does not depend on A .

Supposing that $R_A = A^{1/3}$, i.e., that the range of the ΛA potential, R_Λ^0 , is proportional to the radius of the core A_Z , we get $\Delta B_{\Lambda\Lambda} \sim A^{-1}$, while supposing that $R_A = A^{1/6}$, i.e., that R_Λ^0 is proportional to the nucleon oscillator radius of the oscillator shell model, we get $\Delta B_{\Lambda\Lambda} \sim A^{-1/2}$. The functions $\Delta B_{\Lambda\Lambda} = \text{const} \cdot A^{-1/2}$ and $\Delta B_{\Lambda\Lambda} = \text{const} \cdot A^{-1}$ are presented on fig. 1 by the solid and the dot-dashed lines, respectively, together with the function $\Delta B_{\Lambda\Lambda} = \text{const} \cdot A^{-1/3}$ (the dashed line). It is seen from the fig. 1, that the slope of the $\Delta B_{\Lambda\Lambda}$ A -dependence obtained in Skyrme-Hartree-Fock calculations for medium $\Lambda\Lambda$ hypernuclei is close to the one of the function $\Delta B_{\Lambda\Lambda} = \text{const} \cdot A^{-1}$, while for heavy $\Lambda\Lambda$ hypernuclei the slope is closer to the one of the function $\Delta B_{\Lambda\Lambda} = \text{const} \cdot A^{-1/3}$ for the $\Delta B_{\Lambda\Lambda}$ values obtained with the S $\Lambda\Lambda$ 1 and S $\Lambda\Lambda$ 2 interactions and to the slope of the function $\Delta B_{\Lambda\Lambda} = \text{const} \cdot A^{-1/2}$ for $\Delta B_{\Lambda\Lambda}$ values obtained with the S $\Lambda\Lambda$ 3 interaction. The slope of the $\Delta B_{\Lambda\Lambda}$ values obtained in the three-body approach shows the behavior that is close to $\Delta B_{\Lambda\Lambda} \sim A^{-1/2}$.

However, the differences between various $\Delta B_{\Lambda\Lambda}$ A -dependencies are not well-pronounced, and the main conclusion that we derive from the fig. 1 is that very different models and very different interactions yield more or less the same generally decreasing $\Delta B_{\Lambda\Lambda}(A)$ behavior. At the same time, the effects of the strong core polarization cause deviations from the general trend that are seen from the fig. 2 to be unimportant in the heavy mass region.

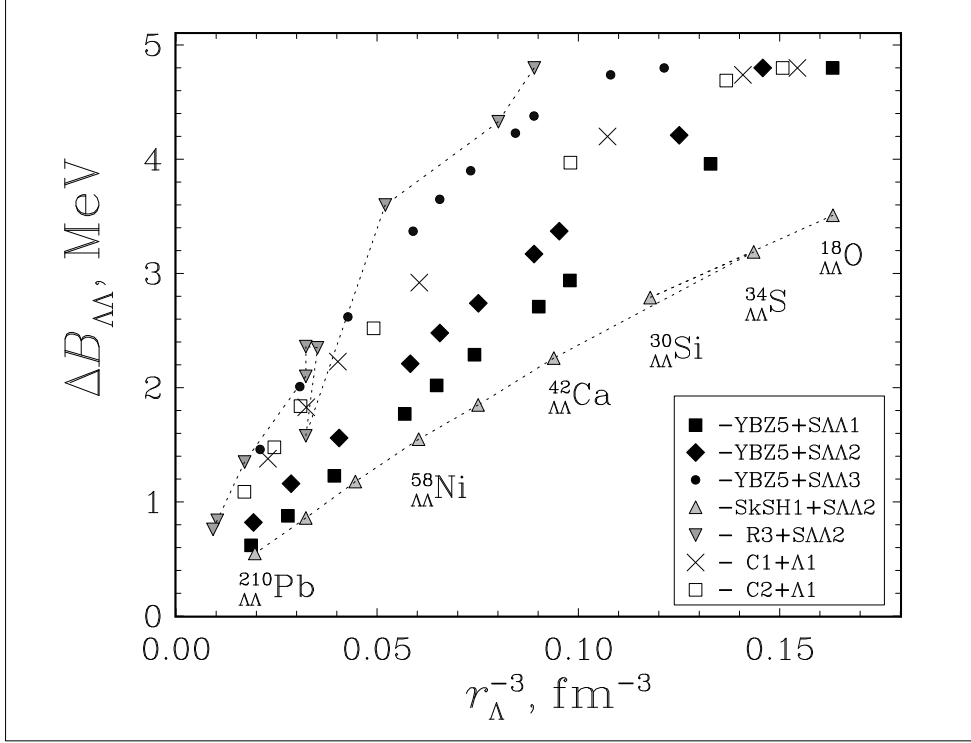


Figure 3: The $\Delta B_{\Lambda\Lambda}$ scaling with r_{Λ}^{-3} for various interactions. See fig. 1 and 2 for details.

As the rms radius of Λ hyperons, r_{Λ} , in heavy $\Lambda\Lambda$ hypernuclei is proportional to R_A , $r_{\Lambda} \sim R_A$, the scaling property (11) means that

$$\Delta B_{\Lambda\Lambda} \sim r_{\Lambda}^{-3}. \quad (12)$$

The plot of $\Delta B_{\Lambda\Lambda}$ vs r_{Λ}^{-3} is presented on fig. 3. The scaling (12) seems to be universal. The irregularities in the $\Delta B_{\Lambda\Lambda}$ behavior obtained with the strongly polarizing potentials are considerably smoothed in the r_{Λ}^{-3} -dependence. In the case of SkSH1 interaction, the oscillation of the $\Delta B_{\Lambda\Lambda}(A)$ dependence of fig. 2 manifests itself on fig. 3 only as an inverse order of the points corresponding to $^{30}_{\Lambda\Lambda}\text{Si}$ and $^{34}_{\Lambda\Lambda}\text{S}$ lying nearly on the same line with all other points. Deviations from the single line of the points corresponding to R3 interaction are more pronounced, but they are much smaller than on fig. 2.

In the heavy-mass (small r_{Λ}^{-3}) region the scaling (12) appears to be very accurate. All the $\Delta B_{\Lambda\Lambda}(r_{\Lambda}^{-3})$ dependencies obtained in our calculations behave according to (12) in this region, i.e., the $\Delta B_{\Lambda\Lambda}$ values for heavy $\Lambda\Lambda$ hypernuclei obtained in the same way lie on a straight line on the plane $\Delta B_{\Lambda\Lambda}$ vs r_{Λ}^{-3} . The deviation from the straight line in the $\Delta B_{\Lambda\Lambda}(r_{\Lambda}^{-3})$ dependence in light and medium mass $\Lambda\Lambda$ hypernuclei increases with the range of the $\Lambda\Lambda$ interaction. Note, that the SAA1 and SAA2 Skyrme-Hartree-Fock results presented on fig. 3 show the linear

behavior even in the light-mass region, while the S $\Lambda\Lambda$ 3 Skyrme-Hartree-Fock results that correspond to the much larger range of the $\Lambda\Lambda$ interaction, deviate from the straight line in the medium-mass region. The steepness of the lines increases with the strength of the $\Lambda\Lambda$ interaction. Note, that the points representing the results of the three-body calculations with the same $\Lambda\Lambda$ interaction lie on the same curve.

Concluding, we have calculated A -dependence of the bond energy of Λ hyperons in the $\Lambda\Lambda$ hypernuclear ground states. This dependence is shown to be a generally decreasing function. The $\Lambda\Lambda$ interaction cannot be extracted from the information about the bond energies only even if they will be measured in a wide range of A . On the other hand, the $\Delta B_{\Lambda\Lambda}$ A -dependence appears to be closely connected with the A -dependence of the rms radius of the hyperon orbital. The later has been calculated for single- Λ hypernuclei in various approaches [23, 24, 25, 26]. However, no reliable information on r_Λ has been extracted from any available data up to now. It should be noted that r_Λ 's calculated by us are not directly related with the rms radii in single- Λ hypernuclei due to some perturbation induced by the 2nd hyperon. Nevertheless, this perturbation is not strong and does not change the general trend. Therefore, the information on the rms radii of the hyperon orbitals in Λ hypernuclei is very important for deducing the $\Lambda\Lambda$ interaction parameters from $\Lambda\Lambda$ hypernuclear data.

Among the problems connected with the $\Lambda\Lambda$ hypernuclear physics, the possible existence of H dibaryon is of a great interest. It is known, that the existence of $\Lambda\Lambda$ hypernuclei restricts the feasibility of long-living H dibaryon, although does not rejects it completely [27, 28]. On the other hand, up to now only three objects have been observed that are conventionally identified as $\Lambda\Lambda$ hypernuclei, and the corresponding experimental data can be alternatively reinterpreted as H hypernuclei [29]. From this point of view, it seems to be interesting to study the A -dependence of the bond energy for H hypernuclei, too, that can possibly serve for the discrimination of the above alternatives.

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